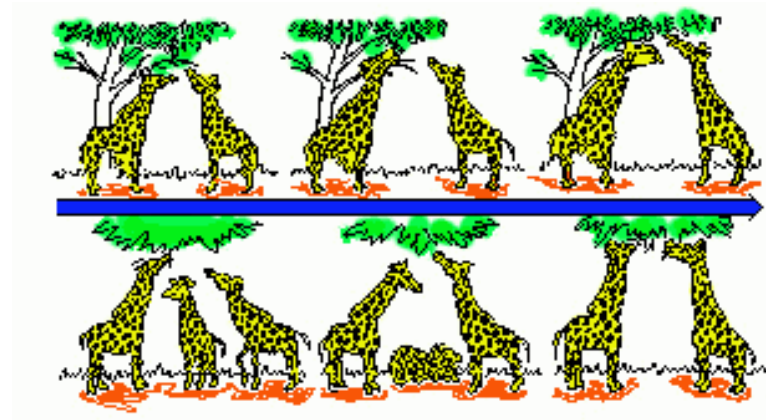




PSO アルゴリズム

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最適値問題を解く

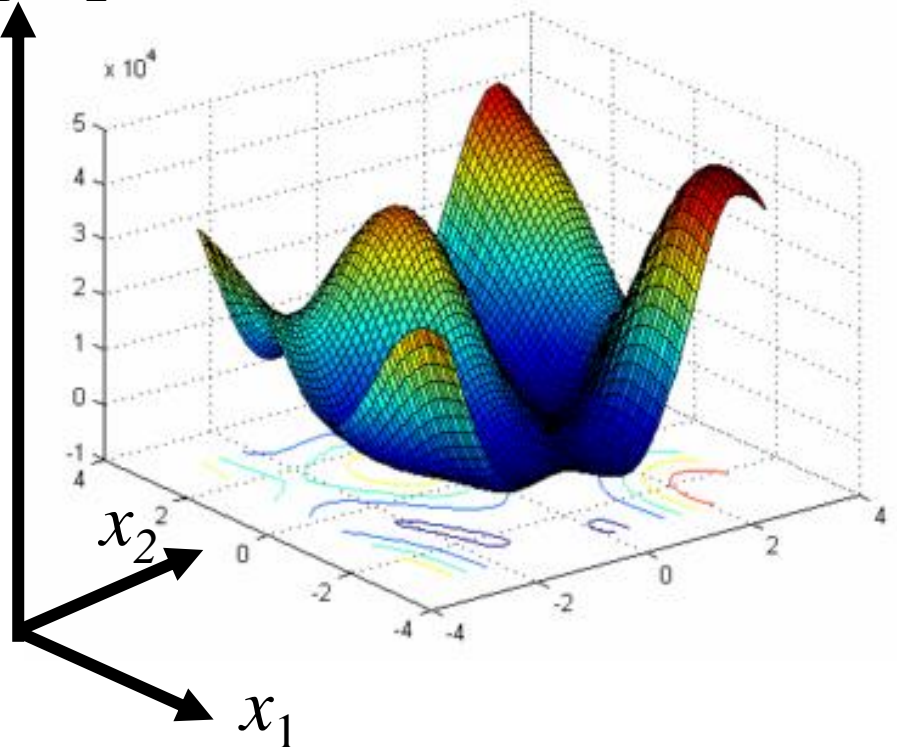
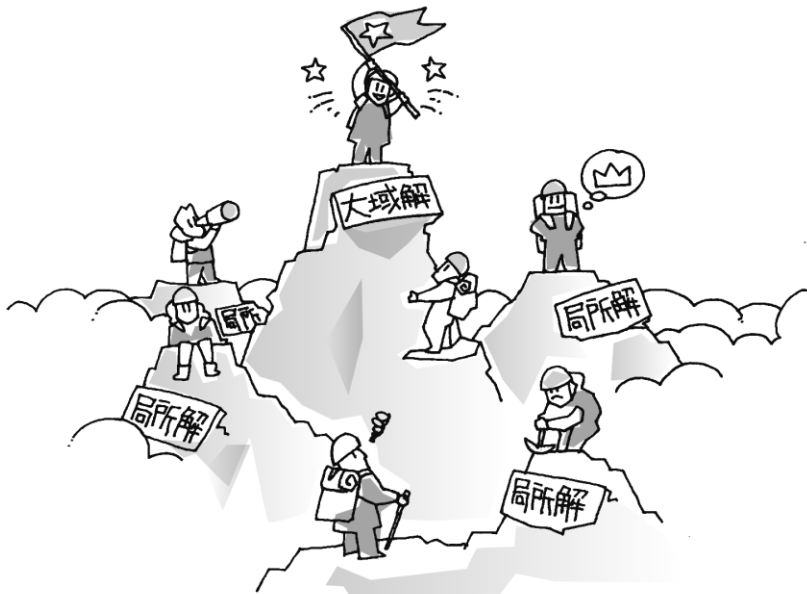
- 最大値を求める問題

$x \in X$ 内で $\max_x \{f(x)\}$ を与える x を求めよ。

- 工学やAIの最も基本的探索問題

$f(x_1, x_2)$

適合度ランドスケープ



Particle Swarm Optimization

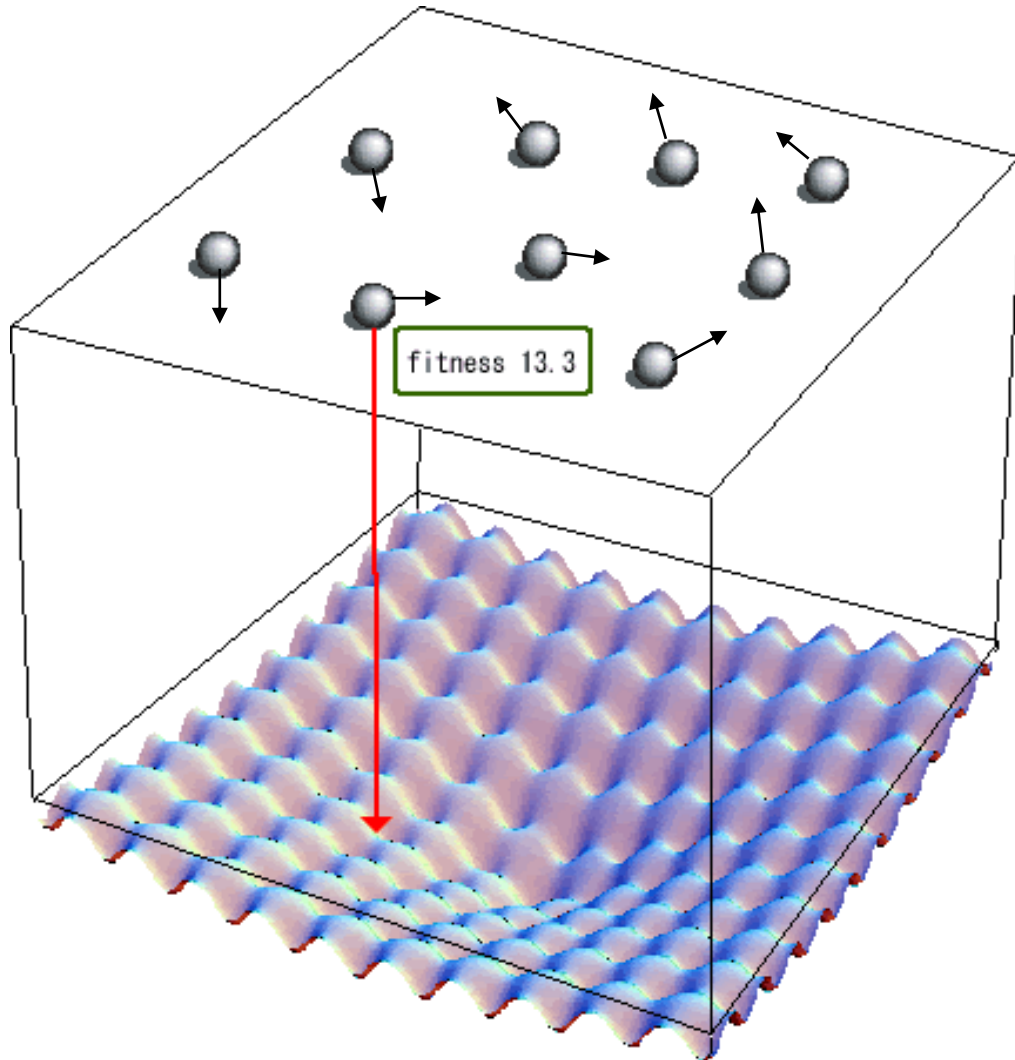
- James Kennedy and Russel C. Eberhart(1995)

```
begin
  initialize
  while(not terminate-condition)do
    begin
      evaluate
      calcurale new velocity vectors
      move
    end
  end
end
```

$$\vec{v}_i = \chi(\omega\vec{v}_i + \psi_1(\vec{p}_i - \vec{x}_i) + \psi_2(\vec{p}_g - \vec{x}_i))$$

$$\vec{x}_i = \vec{x}_i + \vec{v}_i$$



PSO Algorithm

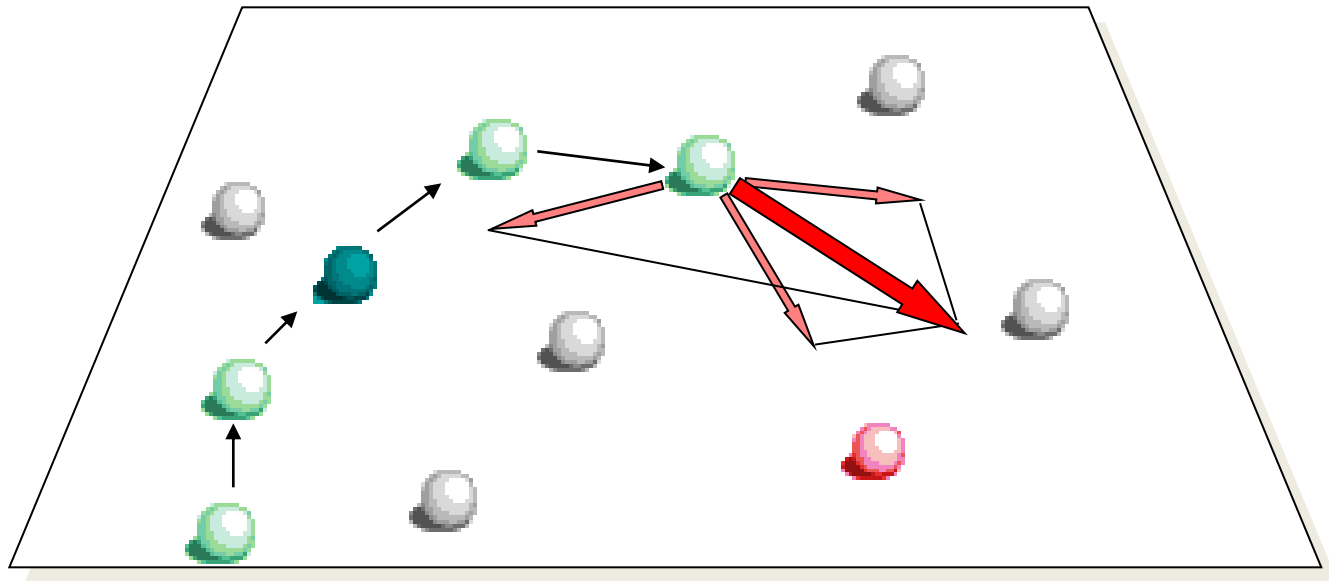


- PSO consists of a number of particles moving around in the search space, each representing a possible solution to numerical problems

PSO Algorithm

- Particulars about motion of PSO

 : global best
 : local best



generation : T

Particle Swarm Optimization

- Swarm : a set of particles (S)
- Particle: a potential solution
 - Position, $X_i = (x_{i1}, x_{i2}, \dots, x_{in}) \in \mathbb{R}^n$
 - Velocity, $V_i = (v_{i1}, v_{i2}, \dots, v_{in}) \in \mathbb{R}^n$

- Each particle maintains

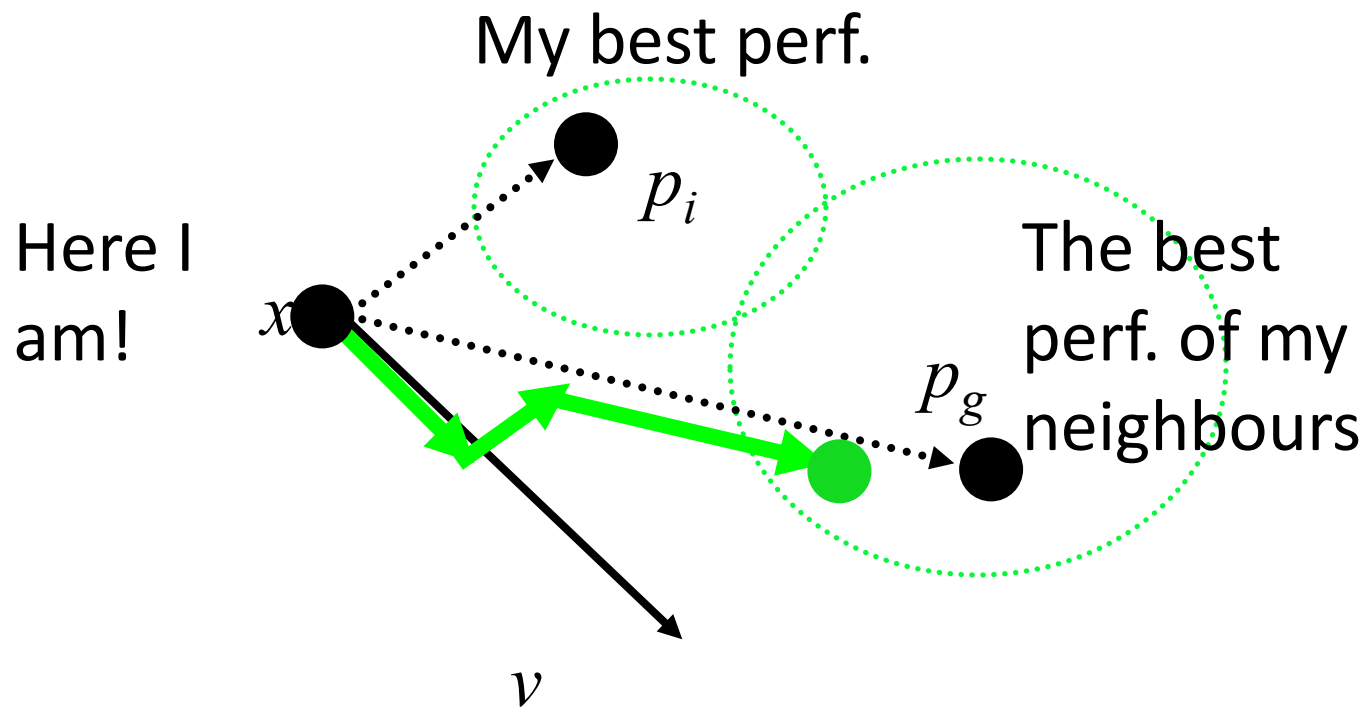
- Individual best position:

$$P_i = (p_{i1}, p_{i2}, \dots, p_{in}) \in \mathbb{R}^n$$
$$pbest_i = f(P_i)$$

- Swarm maintains its global best:

$$P_g \in \mathbb{R}^n$$
$$gbest = f(P_g)$$

pbest and gbest



PSO Algorithm

- Basic algorithm of PSO:
 1. Initialize the swarm from the solution space
 2. Evaluate fitness of each particle
 3. Update individual and global bests
 4. Update velocity and position of each particle
 5. Go to step 2, and repeat until termination condition

PSO Algorithm (cont.)

- **Original velocity update equation:**

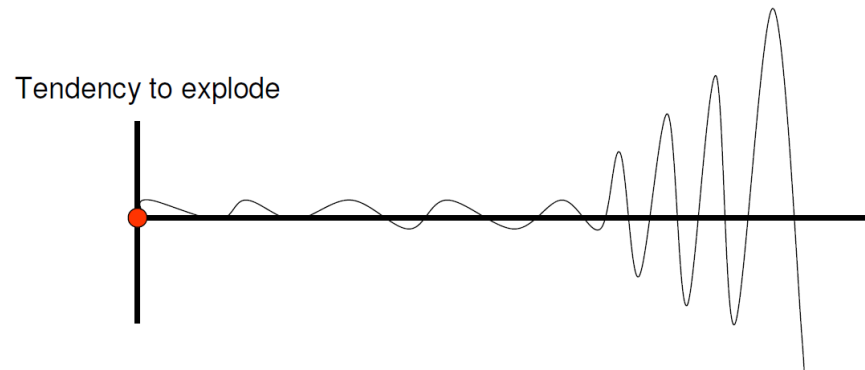
$$V_i^{t+1} = \underbrace{V_i^t}_{\text{Inertia}} + \underbrace{\varphi_1 \cdot r_1 (P_i - X_i^t)}_{\text{Cognitive Component}} + \underbrace{\varphi_2 \cdot r_2 (P_g - X_i^t)}_{\text{Social Component}}$$

– with $r_1, r_2 \sim U(0,1)$

– φ_1, φ_2 : acceleration constant

PSO Algorithm - Parameters

- Acceleration constants φ_1, φ_2
 - Small values limit the movement of the particles
 - Large values : tendency to explode toward infinity
 - In general $\varphi_1 + \varphi_2 \leq 4$



- Maximum velocity
 - Velocity is a stochastic variable => uncontrolled trajectory

If $v_{id} > v_{max}$ then $v_{id} = v_{max}$
else if $v_{id} < -v_{max}$ then $v_{id} = -v_{max}$

Pseudocode

Equation (a)

$$v[] = c0 * v[] \\ + c1 * \text{rand}() * (\text{pbest}[] - \text{present}[]) \\ + c2 * \text{rand}() * (\text{gbest}[] - \text{present}[]) \\ \text{(in the original method, } c0=1, \text{ but many} \\ \text{researchers now play with this parameter)}$$

Equation (b)

$$\text{present}[] = \text{present}[] + v[]$$

Pseudocode

```
For each particle
  Initialize particle
END

Do
  For each particle
    Calculate fitness value
    If the fitness value is better than its personal best
      set current value as the new pBest
    End
  End

  Choose the particle with the best fitness value of all as gBest
  For each particle
    Calculate particle velocity according equation (a)
    Update particle position according equation (b)
  End
While maximum iterations or minimum error criteria is not attained
```

PSOのシミュレータ

